所　　別	組　別	考試科目	考試日期	節　次	備　　　　註
資訊科技與管理研究所	一般生、在職生	離散數學	5月5日	第二節	

Show all your work and write your answers clearly.
You may write the answers in Chinese or English.

1. (10 points) What is the **Minimal Spanning Tree**? Describe a method (either Prim's method or Kruskal's method) in finding a minimal spanning tree. Give two applications how it can be used.

2. (10 points) Use a regular express to state the following definition for a user command:

   A user command (Command) consists of zero or more Command_String terminated by a Delimiter. A command string contains a command name (Name), followed by one or more white spaces (White_Space), followed by zero or more Options, followed by an Argument_List.

3. (10 points) Draw a *non-directed* graph and a *directed* graph of 5 nodes (A, B, C, D, E). Then use adjacency matrixes to define your own graph.

4. (10 points) Draw a tree of your own with vertices labeled. Explain the following definitions with words and from your graph:

   (a) root
   (b) leaf
   (c) parent
   (d) child
   (e) sibling
   (f) level
   (g) height
   (h) path
   (i) ancestor
   (j) descendent

5. (5 points) Show that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

6. (10 points) If $A = \{0, 1\}$, then find $A^2, A^3, A^*, 2^A$.

7. (10 points) In Boolean algebra, let $a \oplus b = (a \wedge \bar{b}) \vee (\bar{a} \wedge b)$. Show the following:

   (a) $a \oplus b = b \oplus a$

   (b) $a \oplus (b \oplus c) = (a \oplus b) \oplus a$

8. (5 points) Find the disjunctive normal form (DNF) for the Boolean expression:

$$(\overline{a \wedge b \vee \bar{a} \wedge c})(\overline{a \vee (b \wedge c)})$$

9. (10 points) Is the following expression tautology?

$$(A \wedge B \rightarrow C) \leftrightarrow (A \wedge \bar{C} \rightarrow \bar{B})$$

10. (5 points) Simplify the following Boolean expression:

$$x_1 x_2 x_3 x_4 + x_1 x_2' x_3 x_4 + x_1' x_2' x_3 x_4 + x_1 x_2 x_3 x_4' + x_1 x_2' x_3 x_4' + x_1 x_2' x_3' x_4'$$

11. (5 points) Draw the state diagram (state graph) from the following state table:

PresentState	NextState PresentInput 00	01	10	11	Output
S_0	S_0	S_1	S_2	S_1	0
S_1	S_2	S_1	S_3	S_2	1
S_2	S_0	S_2	S_2	S_3	1
S_3	S_2	S_1	S_3	S_1	0

12. (10 points) Draw a state graph to describe the following problem:

A vending machine sells a drink (NT\$15). It accepts three conins: NT\$1, NT\$5, NT\$10. It dispenses a drink when it receives enough money. It also gives changes when more than NT\$15 is given.