※ 選擇題，共二十題，每題五分
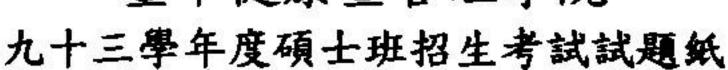
1. How many distinct nonnegative integer solution are there to the equation
   $x_1 + x_2 + x_3 = 7$ in which $x_1 \geq 3$?

   (A) 36 (B) 15 (C) 84 (D) none of the above.

2. Let $S = \{2, 4, 6, 8\}$ and $T = \{1, 5, 7\}$. Find the number of subjective functions
   from S to T.　(A) 64 (B) 81 (C) 36 (D) none of the above.

3. If all the nodes of a simple, connected, planar graph have degree 4 and the number
   of edges is 12, into how many regions does it divide the plane?

   (A) 8 (B) 9 (C) 10 (D) none of the above.

4. How many positive integer n less than 6000 satisfy gcd(n, 6000)=1, where gcd(x, y)
   denotes the greatest common divisor of x and y?

   (A) 3200 (B) 1600 (C) 800 (D) none of the above.

5. What is the value of $\sum_{k=1}^{9} k \, 3^k$?

   (A) 259590 (B) 250995 (C) 250959 (D) none of the above.

6. Find the determinant of the matrix $\begin{bmatrix} 1 & 3 & 2 & 1 \\ 2 & 5 & 1 & 0 \\ 0 & 1 & 2 & 5 \\ 1 & 0 & 1 & 1 \end{bmatrix}$.

   (A) 28 (B) 29 (C) 30 (D) none of the above.

7. Let $S = \{1, 2, 3, 4\}$.　Which of the following properties holds for
   relation $R = \{(1,2), (2,2), (2,4), (3,1), (4,2)\}$?

   (A) reflexivity (B) antisymmetry (C) transitivity (D) none of the above.

8. Let x_n be a sequence satisfying $x_n = -2n \, x_{n-1} + 3n(n-1) \, x_{n-2}$ with $x_0 = 1$,
   $x_1 = 2$, what is x_n?
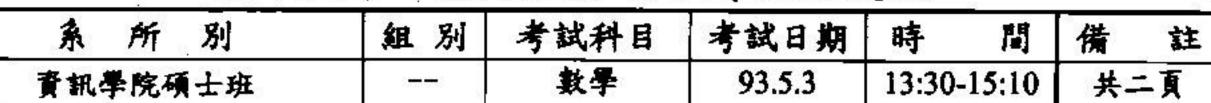
   (A) $x_n = 2^n$ (B) $x_n = (\frac{2}{9} - \frac{15n}{18})(-2)^n + \frac{7}{9}$ (C) $x_n = \frac{n!}{4}(5 - (-3)^n)$
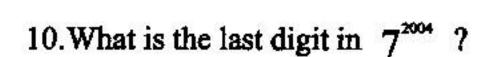
   (D) none of the above.

9. Which of the following is incorrect?

   (A) $K_{2,3}$ is a planar graph　　(B) Petersen graph is a planar graph

   (C) $K_{3,3}$ is not a planar graph　(D) K_4 is a planar graph.

10. What is the last digit in 7^{2004} ?

    (A) 7 (B) 3 (C) 1 (D) none of the above.

11. Which of the following statement is tautology?

    (A) $[\bar{B} \wedge (A \to B)] \to \bar{A}$ (B) $\overline{A \vee B} \equiv \bar{A} \vee \bar{B}$ (C) $A \to B \equiv \bar{A} \wedge B$ (D) $(A \to B) \wedge A \to \bar{B}$

12. A collection S of strings of characters is defined recursively by (1). a and b belong to

    S. (2). If x belongs to S, so does xb.   Which of the following belongs to S?

    (A) aba (B) aaab (C) aaaaa (D) none of the above.

13. For $A = \{1, \ 2, \ 3, \ 4, \ 5, \ 6, \ 7, \ 8 \}$, determine the number of subsets of A

    containing three elements.

    (A) 58 (B) 57 (C) 56 (D) none of the above

14. Find the coefficient of x^{16} in $\left(1 + x^4 + x^8\right)^{10}$.

    (A) 615 (B) 620 (C) 645 (D) none of the above.

15. Let $f : R \to R$ be defined by $f(x) = x^n$, where n is a fixed, positive integer. For

    what value of n could f be bijective?

    (A) 2 (B) 3 (C) 4 (D) none of the above.

16. Let $f : R \to R$ be defined by $f(x) = x^2$. Let $g : R \to R$ be defined by

    $g(x) = 3x + 1$, what is the value of $(g \circ f)(4) = ?$

    (A) 49 (B) 51 (C) 48 (D) none of the above.

17. Which of the following characteristics could exist?

    (A) four nodes of degree 1, 2, 3, and 4, respectively.

    (B) simple, four nodes of degree 1, 2, 3, and 4, respectively.

    (C) four nodes of degree 2, 3, 3, and 3, respectively.

    (D) four nodes of degree 2, 2, 2, and 3, respectively.

18. What is the number of all nonisomorphic simple graphs having four vertices?

    (A) 11 (B) 10 (C) 9 (D) none of the above.

19. Which of the following complete graphs could contain an Euler cycle?

    (A) K_{22} (B) K_{24} (C) K_{26} (D) none of the above.

20. Which of the following complete bipartite graphs could contain a Hamiltonian cycle?

    (A) $K_{20,21}$ (B) $K_{20,22}$ (C) $K_{10,30}$ (D) none of the above.